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Set A (2075-04-09) Hints & Solution

Section I

1. (c)

$$KE = \frac{3}{2}KT$$

or, $K = \frac{2E}{3T} = ML^2T^{-2}\theta^{-1}$

2. (a)
3. (b)
4. (c)
5. (b)
6. (c)
7. (a)
8. (d)

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Where $\mu = A + \frac{B}{\lambda^2}$

Here, $\lambda_r > \lambda_v$ so $\mu_r < \mu_v$

$\therefore$ f will be maximum for red

9. (d)

$$\frac{e/m}{2e/4m} = 2 : 1$$

10. (c)
11. (d)

$$\tan\theta = \cot 2\theta = \tan\left(\frac{\pi}{2} - 2\theta\right)$$

or, $\theta = n\pi + \frac{\pi}{2} - 2\theta$

$\therefore \quad \theta = (2n + 1)\frac{\pi}{6}$

12. (b)

$$\tan^{-1}\alpha + \tan^{-1}\beta = \tan^{-1}\left(\frac{\alpha + \beta}{1 - \alpha\beta}\right) = \tan^{-1}\left(\frac{\frac{5}{6}}{1 - \frac{1}{6}}\right)$$

$$= \tan^{-1}1 = \frac{\pi}{4}$$

13. (a)

Comparing to $x^2 + y^2 + 2gx + 2fy + c = 0$

$g^2 = f^2 = c = a^2$

So, touches both axes

14. (c)

$(x - 4)^2 + y^2 < (x - 2)^2 + y^2$

$\Rightarrow \quad -8x + 16 < -4x + 4$

$\Rightarrow \quad 12 < 4x \Rightarrow 3 < x$

$\Rightarrow$ Re(z) > 3

15. (c)

Since parallel support means they have same or opposite direction.

16. (c)

Two planes taken together given a lines so we have three lines

x = 1, y = 2 …..(1)

x = 1, z = 3 …..(2) and y = 2, z = 3

All of them passes through (1, 2, 3)

17. (a)

$$\begin{vmatrix} p & 2 \\ 1 & 1 \end{vmatrix} \neq 0$$

$\Rightarrow \quad p - 2 \neq 0$

$\therefore \quad p \neq 2$

18. (d)

$$\lim_{x\to\infty} \frac{e^x - 1}{x} \quad \left[\frac{\infty}{\infty}\right]$$

$$= \lim_{x\to\infty} e^x$$

$= e^{\infty} = \infty$ (does not exist)

19. (b)

Total no. of attempts = 10 × 10 × 10 = 1000.

There is only one successful and hence no. of unsuccessful attempts = 1000 – 1 = 999

20. (b)

$$\int \frac{1}{\frac{e^x - e^{-x}}{2}} dx = 2\int \frac{e^x}{(e^x)^2 - 1} dx$$

$$= 2\int \frac{dt}{t^2 - 1} = 2.\frac{1}{2}\log\left(\frac{t - 1}{t + 1}\right) + c$$

$$= \log\left(\frac{e^x - 1}{e^x + 1}\right) + c$$

Put $e^x = t$

$e^x = t$

$\therefore \quad e^x\,dx = dt$

21. (c)

1 mole of $H_2O = 3N_A$ atoms

0.1 mole of $H_2O = 0.3N_A$ atoms

22. (c)

O.N. of metal in alloys is O

23. (d)

In CH_3Cl, carbon is surrounded by different atoms so the net dipole is not zero.

24. (b)

He has smallest atomic size so it has highest first ionization energy.

25. (b)
H_3PO_3 in a dibasic acid

26. (b)
HSO_4^- can both donate and accept proton so it is both acid and base

27. (c)
Cl_2 with hot and conc. NaOH gives $NaClO_3$

28. (a)
Hypo is $Na_2S_2O_3.5H_2O$

29. (d)
Na_2SO_4 is not used to remove hardness of water.

30. (b)
If two halogen atoms are bounded to same carbon atom then it is called gem – dihalide.

31. (a)
The addition of HBr to alkene starts with addition of H^+ -ion so it is an electrophilic addition reaction.

32. (c)
The product is benzene which on ozonolysis gives glyoxal

33. (b) 34. (b) 35. (c) 36. (d) 37. (c) 38. (b)
39. (c) 40. (b) 41. (b) 42. (b) 43. (b) 44. (b)
45. (a) 46. (b) 47. (a) 48. (b) 49. (c) 50. (b)
51. (b) 52. (a) 53. (c) 54. (d) 55. (b) 56. (c)
57. (b) 58. (b) 59. (c) 60. (b)

Section – II

61. (b)

$$V_{av} = \frac{2V_1V_2}{V_1 + V_2} = \frac{2 \times 40 \times 60}{40 + 60} = 48 \text{ km/hr}$$

62. (b)

$$\text{Net force experienced} = \frac{\text{Total impulse}}{\text{Total time}} = \frac{m\Delta v}{t}$$
$$= 30N$$

63. (a)
Loss in PE = Gain in KE

$$mgh = \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2$$

$$\text{or, } mgh = \frac{1}{2} \times \frac{2}{5} mr^2 \times \frac{v^2}{r^2} + \frac{1}{2} mv^2$$

$$\text{or, } mgh = \frac{2mv^2 + 5mv^2}{10}$$

$$\text{or, } v = \sqrt{\frac{10}{7} gh}$$

64. (b)

$$\frac{1}{2} mv^2 = \frac{1}{2} kx^2$$

$$\text{or, } x = v\sqrt{\frac{m}{k}} = 1.5\sqrt{\frac{0.5}{50}} = 0.15 \text{ m}$$

65. (d)
Thermal capacity = ms = 40 × 0.2 = 8 cal /°c
= 4.2 × 8 J/°c = 33.6 J/°c

66. (c)

$$\theta_n = \frac{\theta_c + \theta_i}{2} \qquad \therefore \theta_c = 2\theta_n - \theta_I = -20^oc$$

67. (c)

$$\phi = \frac{2\pi x}{\lambda}$$

$$\text{or, } \lambda = \frac{2\pi x}{\phi} = \frac{2\pi \times 0.4}{1.6\pi} = 0.5m$$

$$\therefore \quad \lambda = \frac{v}{\lambda} = 660 \text{ Hz}$$

68. (a)

$$f' = \frac{9}{8} f$$

$$\text{App. Frequency } (f') = f \times \frac{(v + u)}{(v - u)}$$

$$\text{or, } \frac{9}{8} f = f \times \frac{(v + u)}{(v - u)}$$

$$\text{or, } 9v - 9u = 8v + 8u$$

$$\text{or, } v = 17u$$

$$\text{or, } u = \frac{v}{17} = \frac{340}{17} = 20 \text{ m/s}$$

69. (c)
Work done = change in energy

$$= \frac{1}{2}\left(C + \frac{C}{2}\right) V^2 = \frac{3}{4} CV^2$$

70. (d)
$E = V + Ir$
$12 = V + 60 \times 5 \times 10^{-2}$
$\Rightarrow \quad V = 9$ volt

71. (c)

Torque (τ) = $MB \sin\theta = \vec{M} \times \vec{B}$

72. (b)

$$\frac{f_a}{f_l} = \frac{\left(\frac{\mu_g}{\mu_l} - 1\right)}{(\mu_g - 1)}$$

$$\text{or, } \frac{2}{f_l} = \frac{\frac{1.5}{1.25} - 1}{1.5 - 1}$$

$$\text{or, } f_l = 5 \text{ cm}$$

73. (b)

$n = 8, \quad D = 0.72$ cm $\quad R = 300$

$r = \frac{D}{2} = 0.36$ cm

$\lambda = ?$

We have (for transmitted system)

$$\lambda = \frac{2r^2}{(2n-1)R} = \frac{2 \times (0.36)^2}{(2 \times 8 - 1) \times 300}$$

$$= 5760 \times 10^{-8} \text{ cm}$$

$\therefore \quad \lambda = 5760 \times 10^{-10}$ m

74. (a)

$$KE = hf - \phi = \frac{hc}{\lambda} - \phi$$

$$= \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{5000 \times 10^{-10} \times 1.6 \times 10^{-19}} - 1.9$$

$= 2.48 - 1.9 = 0.58$ ev

75. (d)

No. of half lives, $n = \frac{t}{T} = \frac{6400}{800} = 8$

$\therefore \quad \frac{N}{N_o} = \left(\frac{1}{2}\right)^8 = \frac{1}{256}$

76. (b)

$$a\cos^2\frac{B}{2} + b\cos^2\frac{A}{2} = a\left(\frac{1+\cos B}{2}\right) + b\left(\frac{1+\cos A}{2}\right)$$

$$= \frac{a + a\cos B + b\cos A + b}{2}$$

$$= \frac{a+b+c}{2}$$

77. (b)

Put $x = \tan\theta$

$$\sin\left\{\tan^{-1}\left(\frac{1-\tan^2\theta}{2\tan\theta}\right) + \cot^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)\right\}$$

$$= \sin\left\{\tan^{-1}\tan\left(\frac{\pi}{2} - 2\theta\right) + \cos^{-1}\cos 2\theta\right\}$$

$$= \sin\left(\frac{\pi}{2} - 2\theta + 2\theta\right) = 1$$

78. (b)

$$\left(\frac{x^2}{1!} + \frac{x^4}{2!} + \frac{x^6}{3!} + \ldots.\right) - \left(\frac{y^2}{1!} + \frac{y^4}{2!} + \frac{y^6}{5!} + \ldots.\right)$$

$= (e^{x^2} - 1) - (e^{y^2} - 1)$

$= e^{x^2} - e^{y^2}$

79. (d)

Normal is $x + 2y = k$, which passes through centre (1, 0) of circle. So, $k = 1$

$\therefore \quad x + 2y = 1$

80. (d)

For $(x-2)^2 = y - 1$

Length of LR = 1

So, (d) is incorrect

81. (a)

$$\begin{vmatrix} 1 & \frac{\log y}{\log x} & \frac{\log z}{\log x} \\ \frac{\log x}{\log y} & 1 & \frac{\log z}{\log y} \\ \frac{\log x}{\log z} & \frac{\log y}{\log z} & 1 \end{vmatrix}$$

$$= \frac{1}{\log x \log y \log z} \begin{vmatrix} \log x & \log y & \log z \\ \log x & \log y & \log z \\ \log x & \log y & \log z \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$= 0$

82. (d)

Planes,

$4x - 4y + 2z + 2 = 0$ and $4x - 4y + 2z + 3 = 0$

$$\text{Distance} = \frac{3-2}{\sqrt{16+16+4}} = \frac{1}{6}$$

83. (b)

$$f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$$

$f(t) = t^2 - 2$

$\therefore \quad f(x) = x^2 - 2$

84. (a)

$\vec{BC} = \vec{i} + \vec{j}, \quad \vec{AB} = \vec{i} - \vec{j}$

$\vec{AC} = \vec{AB} + \vec{BC} = \vec{i} - \vec{j} + \vec{i} + \vec{j} = 2\vec{i}$

85. (c)

$f'(x) = \frac{1}{2} - \frac{2}{x^2} \qquad \therefore f''(x) = \frac{4}{x^3}$

$f'(x) = 0 \Rightarrow x^2 = 4 \quad \therefore \; x = -2, 2$

For $x = 2, \quad f''(x) > 0$

So, $x = 2$ point of local minima

86. (a)

$$y = \log_{\sqrt{e}}(\sin x) = \frac{1}{1/2}\log_e \sin x = 2\log_e \sin x$$

$$\therefore \quad \frac{dy}{dx} = 2.\frac{1}{\sin x}\cos x = 2\cot x$$

87. (d)

$$\int \frac{x\,dx}{\sqrt{4 - x^4}}\,dx = \frac{1}{2}\int \frac{1\,d(x^2)}{\sqrt{4 - (x^2)^2}}$$

$= \frac{1}{2}\sin^{-1}\left(\frac{x^2}{2}\right) + c$

88. (d)

Solving, $x^2 = 2 - x^2$

or, $x^2 = 1$

$\therefore \quad x = \pm 1$

$A = \int_{-1}^{1} (2 - x^2 - x^2)\, dx$

$= \left[2x - \frac{2x^3}{3}\right]_{-1}^{1} = \frac{8}{3}$

89. (d)

Sum $= 1 + 2.2x + 3.(2x)^2 + 4.(2x)^3 + \ldots.$

$= \frac{1}{1-2x} + \frac{1.2x}{(1-2x)^2} = \frac{1}{(1-2x)^2}$

90. (c)

$3(x-1)^2 + 4(y+1)^2 = 5 + 3 + 4 = 12$

$\therefore \quad \frac{(x-1)^2}{4} + \frac{(y+1)^2}{3} = 1$

$\therefore$ Centre (T),

$c = \sqrt{1 - \frac{3}{4}}$ (T)

foci $(1 \pm \sqrt{4-3}, -1) = (1 \pm 1, -1)$

$\Rightarrow$ (2, –1) and (0, –1)

So, (c) is not correct.

91. (c)

$S_2 = \frac{V_1 \times S_1}{V_2} = \frac{50 \times 24}{100} = 12\ N = 6\ M$

92. (d)

$Cu^{++} + 2e \longrightarrow Cu$

1mole 2 mole $= 2 \times 96500$ c

0.3 mole $0.3 \times 2 \times 96500$ c

$= 6 \times 96500$ c

93. (c)

$pOH = -\log[OH^-] = -\log 0.015 = 1.82$

$pH = 14 - 1.82 = 12.18$

94. (b)

$NH_3 + 3Cl_2 \longrightarrow NCl_3 + 3HCl$

95. (c) 96. (c) 97. (b) 98. (c) 99. (d) 100. (d)

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Set B (2075-04-09) Hints & Solution

Section I

1. (c)
   $v = \sqrt{v_x^2 + v_y^2}$
   At highest point $v_y = 0$ so
   $v = v_x$
2. (a)
3. (d)
   Couple always produce rotational motion only
4. (d)
5. (b)
6. (a)
   In equipotential surface work done is 0.
7. (d)
8. (a)
   Can see distant object clearly & unable to see near object.
9. (a)
   de-Broglie's wave length $\lambda = \frac{h}{p}$
   or, $p = \frac{h}{\lambda}$
10. (b)
11. (b)
    $\tan^{-1}a - \tan^{-1}b + \tan^{-1}b - \tan^{-1}c + \tan^{-1}c - \tan^{-1}a = 0$
12. (b)
    One root of $x^2 + ax + 12 = 0$ is 4
    or, $16 + 4a + 12 = 0$
    $\therefore \quad a = -7$
    For (ii), $x^2 - 7a + 6 = 0$ has equal roots
    or, $49 - 4b = 0$
    $\therefore \quad b = \frac{49}{4}$
13. (d)
    Circles touches each other so
    Common tangent $S_1 - S_2 = 0$
    $\therefore \quad -64 + 10x - 16 = 0$
    $x = 8$
14. (a)
    $1 = (1 - z) + z$
    $|1| = |(1 - z)\, z| \le |1 - z| + |z|$
    $\therefore \quad |1 - z| + |z| \ge 1$
    $\therefore$ Minimum value is 1
15. (c)
    For non-zero and non-collinear vectors $\vec{a}$ and $\vec{b}$
    St. $x\vec{a} + y\vec{b} = \vec{0} \Rightarrow x = y = 0$
16. (b)
    The projection of line along x-axis
    $(3 - 1).1 + (4 - 2).0 + (7 - 3).0 = 2$
    Similarly on y and z axis be 2 and 4 respectively
17. (b)
    $A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = 2A$
    $A^4 = A^2\, A^2 = (2A)\,(2A) = 2^2A^2 = 2^3.A$
    $\therefore \quad A^{100} = 2^{99}\, A$
18. (d)
    Women can be seated in a circle in 5! ways then men can be seated in 6! ways.
    Total no. of ways = 5! × 6!
19. (b)
    $\int 2^{\sin x}\, d(\sin x) = \frac{2^{\sin x}}{\log_e 2} + c$
    Since,
    $\int a^x\, dx = \frac{a^x}{\log_e a} = 2^{\sin x}.\log_2 e + c$
20. (b)
    $\lim_{x \to 0} \frac{\log(1 + 3x)}{x} \qquad \left[\frac{0}{0}\right]$
    $= \lim_{x \to 0} \frac{\frac{1}{1 + 3x} \times 3}{1}$ [Using L-Hospital's rule]
    $= 3$
21. (b)
    $\frac{W_{O_2}}{W_{Al}} = \frac{E_{O_2}}{E_{Al}}$
    $W_{O_2} = \frac{8}{9} \times 2.7 = 2.4\ g$
22. (c)
    O.N. of S in $H_2S_2O_8$ is +6 although calculate O.N. is +7 but O.N. cannot be more than group no.
23. (b)
    $Mg(OH)_2$ have ionic bond between Mg^{++} and OH^- –ion and covalent bond in OH^- –ion
24. (a)
    Germanium was given name eka-silicon by Mendeleev.
25. (d)
    In redox rxn involving $K_2Cr_2O_7$ O.N. of Cr changes by 6.
26. (c)
    CH_3COO^- is strong conjugate base of weak acid rest are conjugate base of strong acid.

27. (a)
Calgon is sodium hexameta phosphate and has formula is $Na_2\,[Na_4\,(PO_3)_6]$

28. (b)
NH_3 and CO_2 on heating under pressure gives urea.

29. (d)
Mn_2O_7 has highest O.N. of Mn so it is an acidic oxide.

30. (c)

$$CH_3-\underset{\displaystyle Br}{\underset{|}{CH}}-CH_3 \xrightarrow[aloKOH]{} CH_3-CH=CH_2 \xrightarrow[Br]{HBr} CH_3-\underset{\displaystyle Br}{\underset{|}{CH}}-CH_3$$

31. (c)
Benzene and Nitrobenzene can be separated by fractional distillation.

32. (d)
Neopentane has only primary Hydrogen.

$$CH_3-\underset{\displaystyle CH_3}{\underset{|}{\overset{\displaystyle CH_3}{\overset{|}{C}}}}-CH_3$$

33. (b) 34. (b) 35. (c) 36. (b) 37. (b) 38. (c)
39. (c) 40. (b) 41. (a) 42. (b) 43. (a) 44. (b)
45. (a) 46. (b) 47. (d) 48. (b) 49. (d) 50. (a)
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57. (d) 58. (a) 59. (c) 60. (a)

Section – II

61. (b)

Input for 1 s (Q_1) = $\dfrac{10^6 \times 4.2 \times 5}{3600}$ w = 5833 w

$\eta = \dfrac{w}{Q_1} \times 100\%$

or, $w = \dfrac{30 \times 5833}{100} = 1750$ w

62. (b)

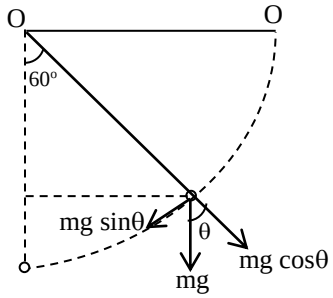
$a = \dfrac{\left(\frac{v}{2}\right)^2 - v^2}{2 \times 40}$

$= \dfrac{O^2 - \left(\frac{v}{2}\right)^2}{2x}$

or, $-\dfrac{3v^2}{4 \times 40} = -\dfrac{v^2}{4x}$

$x = \dfrac{40}{3}$ cm

63. (b)



$mg\sin\theta = ma_t$

$\therefore \quad a_t = \dfrac{\sqrt{3}}{2}\,g$

Now, Loss in PE = Gain in PE

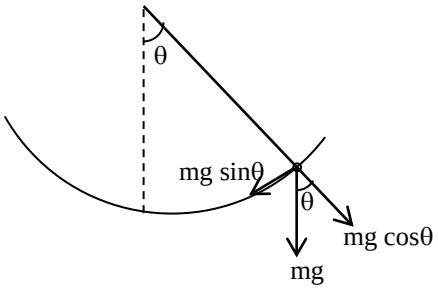
or, $mgl\cos\theta = \dfrac{1}{2}mv^2$

$\therefore \quad v = \sqrt{gl}$

And, $a_c = \dfrac{v^2}{l} = \dfrac{gl}{l} = g$

$\therefore \quad a = \sqrt{a_c^2 + a_t^2} = \dfrac{\sqrt{7}}{2}\,g$

64. (a)



$mg\sin\theta = \mu R = \mu mg\cos\theta$

or, $\tan\theta = \mu$

or, $\dfrac{1}{\cot\theta} = \mu = \dfrac{1}{3}$

$\therefore \quad \theta = \cot^{-1}(3)$

65. (c)

$2.5\,\beta = 2.5\,\dfrac{D\lambda}{d} = \dfrac{2.5 \times 1 \times 6 \times 10^{-7}}{10^{-3}} = 1.5 \times 10^{-3}$m

$= 1.5$ mm

66. (a)

$\mu = \dfrac{\text{Real depth}}{\text{App. depth}}$

$\therefore \quad \text{App. depth} = \dfrac{\text{Real depth}}{\mu} = 4.5$ cm

67. (c)

$KE = \dfrac{hc}{\lambda} - \phi = 1.1$ ev

68. (b)

$$\frac{N'}{N_o} = 1 - \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}} = 2.68 \times 10^{18}\left\{1 - \left(\frac{1}{2}\right)^{3240/1620}\right\}$$

$\therefore \quad N' = 2.01 \times 10^{18}$

69. (d)

$$E = E_3 - E_1 \quad = 3^2\left(-\frac{13.6}{3^2} + \frac{13.6}{1}\right) = 108.8 \text{ ev}$$

70. (a)

$U_1 = U$ (stored energy on capacitor)

$C_{eq} = C'' = 2C$

$$\frac{U_1}{U_2} = \frac{\frac{1}{2}\frac{q^2}{C}}{\frac{1}{2}\frac{q^2}{C'}} = 2$$

$\therefore \quad U_2 = \frac{U}{2}$

71. (d)

Wheat bridge's is balanced, then resistance of galvanometer will be uneffective

$R_{eq} = R$

72. (c)

$I_1 = 10A, \quad I_2 = 0, \quad t = 0.5$ sec

Emf (E) = 220 V

So, $E = -L\frac{dI}{dt} \qquad \therefore L = \frac{220}{20} = 11$ H

73. (b)

$$r = \frac{mv\sin\theta}{Be} = \frac{3 \times 10^5 \sin 30^o}{0.3 \times 10^8}$$

$$= \frac{3 \times 10^5 \times 1}{3 \times 10^7 \times 2} = 0.5 \text{ cm}$$

74. (c)

Loss in PE = Gain in elastic PE

or, $mg(h + r) = \frac{1}{2}kr^2$

or, $4 \times 10\,(3 + r) = \frac{1}{2} \times 500r^2$

or, $250r^2 = 40(3 + r)$

or, $25r^2 - 4r - 12 = 0$

or, $r = \frac{+4 \pm \sqrt{(-4)^2 - 4 \times 25\,(-12)}}{2 \times 25} = \frac{4 \pm 38.8}{50}$

$\therefore \quad r = 0.86$ m

75. (b)

$$\alpha = \frac{\Delta I_c}{\Delta I_e}$$

or, $\alpha = \frac{\Delta I_e - \Delta I_b}{\Delta I_e}$

or, $\Delta I_b = (1 - \alpha)\,\Delta I_e$

$= (1 - 0.95) \times 5$ mA

$= 0.25$ mA

76. (c)

a, b, c are in A.P. then $2b = a + c$

Now,

$$3\tan\frac{A}{2}\cdot\tan\frac{C}{2} = 3\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}\cdot\sqrt{\frac{(s-b)(s-a)}{s(s-c)}}$$

$$= 3\frac{(s-b)}{s} = 3\frac{\left(\frac{a+b+c}{2} - 6\right)}{\frac{a+b+c}{2}} = 3\left(\frac{a+c-b}{a+b+c}\right)$$

$$= 3\left(\frac{2b-b}{3b}\right) = 1$$

77. (a)

$\tan^{-1}(a) + \tan^{-1}(b) = \tan^{-1}3$

$$\tan^{-1}\left(\frac{a+b}{1-ab}\right) = \tan^{-1}3$$

$\therefore \quad a + b = 3 - 3ab = 3(1 - ab)$

Where a, b are non-negative integers

If $a = 0, b = 3$ and $b = 0, a = 3$

$\Rightarrow$ Numbers of positive integral pairs is zero.

Since 0 is not positive integral

78. (d)

$$\text{Given series} = 2\left[x + \frac{1}{3}x^3 + \frac{1}{5}x^5 + \ldots.\right]$$

Where $x = \frac{1}{7}$

$$= \log\left(\frac{1+x}{1-x}\right) = \log\left(\frac{8}{6}\right) = \log_e\left(\frac{4}{3}\right)$$

79. (c)

The equation of tangent is

$$y = mx \pm a\sqrt{1+m^2} = \frac{1}{\sqrt{3}}x \pm 5\sqrt{1+\frac{1}{3}}$$

$$= \frac{x}{\sqrt{3}} \pm \frac{10}{\sqrt{3}}$$

$\therefore \quad x - y\sqrt{3} \pm 10 = 0$

80. (d)

$$(y+2)^2 = -4x + 2 = -4\left(x - \frac{1}{2}\right)$$

$Y^2 = -4X$

$\therefore$ Directrix of parabola: $X = 1$

or, $x - \frac{1}{2} = 1$

$\therefore \quad x = \frac{3}{2}$

Set B (2075-04-09) Hints & Solution

81. (a)

$9(x^2 + 8x) - 16(y^2 + 2y) = 16$

or, $9(x + 4)^2 - 16(y + 1)^2 = 16 + 144 - 16 = 144$

$\therefore \quad \frac{(x+4)^2}{16} - \frac{(y+1)^2}{9} = 1$

Length of LR $= 2 \times \frac{9}{4} = \frac{9}{2}$

82. (a)

d.r's of line $x^2 + y^2 = 0$ are 0, 0, 1 sine in 3D

$x^2 + y^2 = 0$ is equation of z-axis.

So, $\sin\theta = \frac{1.0 + 3.0 + 0.1}{\sqrt{10}} = 0$

$\therefore \quad \theta = 0$

83. (a)

$f(x)\, f(y) - \frac{1}{2}\left[f\left(\frac{x}{y}\right) + f(xy) \right]$

$= \cos(\log(x) \cos(\log y) - \frac{1}{2}[\cos(\log x - \log y) +$

$\cos(\log x + \log y))$

$= \cos(\log x).\cos(\log y) - \frac{1}{2}[2\cos(\log x).\cos(\log y)] = 0$

84. (b)

$|\vec{a}.\vec{b}| = 3 \Rightarrow ab\cos\theta = 3$

$|\vec{a} \times \vec{b}| = 4 \Rightarrow ab\sin\theta = 4$

or, $\tan\theta = \frac{4}{3}$

or, $\cos\theta = \frac{3}{5}$

$\therefore \quad \theta = \cos^{-1}\left(\frac{3}{5}\right)$

85. (b)

$f'(x) = x[4 + 4.2^2.x^2 + \ldots + 20.2^{10}x^{18}]$

or, $f'(x) = 0 \Rightarrow x = 0$ only and $f''(0) > 0$

$\therefore$ Exactly one minimum

86. (c)

$f(g(x)) = I(x) = x \qquad f'(b).2 = 1$

or, $f'(g(x)).g'(x) = 1 \qquad f'(b) = \frac{1}{2}$

$\therefore \quad f'(g(a)).g'(a) = 1$

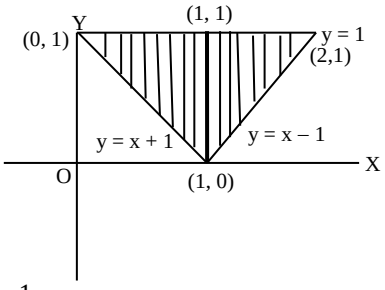
87. (a)

$$\int \frac{1 + \frac{1}{x^2}}{x^2 + \frac{1}{x^2}}\,dx = \int \frac{d\left(x - \frac{1}{x}\right)}{\left(x - \frac{1}{x}\right)^2 + 2}$$

$$= \frac{1}{\sqrt{2}} \cdot \tan^{-1}\left(\frac{x - \frac{1}{x}}{\sqrt{2}}\right) + c$$

$$= \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x^2 - 1}{\sqrt{2}\,x}\right) + c$$

88. (a)



Area $= 2 \times \frac{1}{2} \times 1 \times 1 = 1$

89. (c)

Let a and b be two quantities

Then, $G = \sqrt{ab}$, $\quad A_1 = \frac{2a + b}{3} \qquad A_2 = \frac{a + 2b}{3}$

or, $2A_1 - A_2 = a, \qquad 2A_2 - A_1 = b$

$\therefore \quad$ L.H.S. $= ab = G^2$

90. (a)

Operating $R_1 \to R_1 + R_3$

$$\begin{vmatrix} 1-i & \omega+\omega^2 & \omega^2-\omega^3 \\ i-i & -1 & \omega^2-1 \\ -i & -i+\omega-1 & -1 \end{vmatrix} = \begin{vmatrix} 1-i & -1 & \omega^2-1 \\ 1-i & -1 & \omega^2-1 \\ -i & -i+\omega-1 & -1 \end{vmatrix}$$

$= 0 \quad [R_1 = R_2]$

91. (a)

$N_{mix} = \frac{20 \times 1 - 30 \times 0.5}{20 + 30} = 0.1$ N (w.r.t HCl)

92. (a)

$pH = -\log[H^+] = -\log 0.01 = 2$

93. (c)

1 gram equivalent of any element requires 1F charge. Hence 4 gram equivalents require 4F charge.

94. (d)

H_2S gives black PPt PbS with lead acetate

$(CH_3COO)_2\,Pb + H_2S \to PbS\downarrow + CH_3COOH$

black ppt

95. (d) 96. (a) 97. (b) 98. (b) 99. (c) 100. (b)